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Experiments on the onset of instability in unsteady circular Couette flow

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Experiments have been performed to study the hydrodynamic stability of unsteady circular Couette flow generated by monotonic time-dependent inner-cylinder motions. The onset of instability was determined by measuring the axial component of velocity using laser-Doppler velocimetry; deviations from the pure-swirl value of zero are indicative of the initiation of Taylor vortices. The measurement technique was found to have an 'intrusive' effect on the flow stability, which was eliminated by the design of the experimental procedure. Significant enhancement of stability was found, in qualitative, but not quantitative, agreement with earlier theoretical predictions.

1. Introduction

The hydrodynamic stability of time-dependent flows has attracted an increasing amount of attention in recent years. Since such flows are common, resulting from either start-up or changing operating conditions, a knowledge of their stability properties may have important practical implications. The analytical difficulties involved in the application of linear-stability theory to such flows have been recognized for some time (Shen 1961). For a general, unsteady basic state the governing equations are partial differential equations in both time and space, so that the extraction of information on the behaviour of disturbances is difficult. Moreover, it is bothersome that there is no obvious instability criterion (Homsy 1973). These problems do not arise in the special case of time-periodic basic states (Davis 1976). For uniformly accelerated flows Homsy has demonstrated that the analytical situation can be improved by the application of energy-stability theory.

For the special class of unsteady flows which can be classified as *slowly varying*, the WKBJ approximation can be reasonably used to determine (linear) behaviour of a disturbance. An example from this class, slowly varying circular Couette flow, has been studied by Eagles (1977). He considered the specific case in which the inner-cylinder angular speed varied on a timescale slow compared with the diffusive scale on which disturbances grow. By means of the WKBJ method he determined the growth characteristics of the disturbance kinetic energy and suggested two instability criteria: the first associates instability with a positive growth rate of this energy; while the second uses a relative measure in which its growth is compared with that of the kinetic energy of the basic state. Eagles argues in favour of the former on the basis that the assumed disturbance structure (Taylor vortex) is qualitatively different from the basic state of pure swirl. For several increasing inner-cylinder

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angular speeds his predictions yield the interesting result that stability is enhanced relative to steady Couette flow, i.e. the instantaneous speed at which disturbance growth occurs is larger than that at which steady Couette flow becomes unstable. However, the *amount* of enhancement shows the expected dependence on the choice for the instability criterion, with the absolute criterion yielding less enhancement.

By using Eagles' (1977) method and an initial-value (IV) method (Chen & Kirchner 1971), Chen, Neitzel & Jankowski (1985) obtained results on the influence of initial conditions for a nonlinearly increasing, inner-cylinder angular speed. Their results also show a significant dependence on the instability criteria; in particular a marginal-stability criterion, which allows the disturbance kinetic energy to grow to its initial amplitude before the flow is considered unstable, gave onset times much longer than those based on either of Eagles' criteria. However, for the same criteria, the enhancements from the IV and WKBJ methods do not agree for the complete range of initial conditions considered.

In contrast to linear-stability theory, the application of energy-stability theory to unsteady flows does not pose a serious problem since time appears merely as a parameter in its governing equations. Recently, Neitzel (1982) applied energy-stability theory to several unsteady Couette flows. While the restriction to slowly varying flow was not necessary, two of the cases studied by Eagles (1977) were included. The results show that the growth of a disturbance of *arbitrary size* cannot occur until a critical time, and hence a critical speed, is reached. For certain starting conditions and inner-cylinder accelerations, this speed was *above* the critical speed predicted by linear theory for steady circular Couette flow. Therefore at least some of the enhancement predicted by Eagles must be observable in an experiment.

Experiments on the stability of time-dependent circular Couette flow are few in number and none seem suitable to test the analytical predictions of Eagles (1977). The celebrated experiments of Donnelly (1964) with sinusoidal inner-cylinder motion have motivated a considerable body of work on the stability of time-periodic flows (Davis 1976). Kirchner & Chen (1970) performed flow-visualization experiments on circular Couette flow with an impulsively started inner cylinder. Similar experiments, with rapid linear ramping of the inner-cylinder speed, were performed by Chen, Liu & Skok (1973); their accelerations were too large for the flow to be considered as slowly varying, and instability usually occurred after the final inner-cylinder speed had been reached. Burkhalter & Koschmieder (1974) also considered impulsive starts, but concentrated on the Taylor-vortex equilibrium structure. Park & Donnelly (1981), in an experiment 'not designed to locate absolute onset speeds', studied the influence of a 'slow' inner-cylinder acceleration on the development of a Taylor-vortex structure in the presence of end effects. The relation between inner-cylinder acceleration and the maintenance of 'quasistatic' conditions in a finite geometry has been studied by Park, Crawford & Donnelly (1981); their critical Reynolds number for zero acceleration is somewhat greater than the infinite-geometry prediction.

The purpose of this paper is to report on a series of quantitative experiments designed to study the instability of circular Couette flow for various monotonic time-dependent inner-cylinder motions. These motions were selected so that instability always occurred while the inner cylinder was accelerating. Cases treated by Eagles (1977) and Chen *et al.* (1985) are included in order to provide a comparison with their predicted enhancements. The experimental conditions for instability were based on time-dependent measurements, using laser-Doppler velocimetry, of the axial component of velocity at various positions in a large-aspect-ratio, Taylor–Couette apparatus. Although the measuring device was found to influence these conditions, it was possible to design an experimental procedure that provided measurement-independent onset times.

Substantial effort is currently being spent on the observation and understanding of phenomena associated with finite geometry in Taylor–Couette experiments (e.g. Park & Donnelly 1981; Benjamin & Mullin 1982; Ahlers & Cannell 1983). Particularly dramatic are the bifurcation effects observed in small-aspect-ratio geometries (Mullin 1982). Until recently most analytical studies of the stability of circular Couette flow have assumed that the cylinders are infinitely long (as do the analyses of Eagles 1977, Neitzel 1982 and Chen *et al.* 1985). It is now recognized that the Taylor-vortex structure which is *ultimately* observed in an experiment is influenced by the conditions which exist at the ends of the apparatus (e.g. Park & Donnelly 1981; Park *et al.* 1981). Fixed ends are also known to affect the *development* of the final equilibrium structure by means of a vortex front that originates at the endwall Ekman vortex and propagates into the interior of the annulus. This continuous phenomenon, which is different from the abrupt onset of instability owing to the growth of small disturbances, has been studied experimentally by Ahlers & Cannell (1983) and numerically by Neitzel (1984) and Lücke, Mihelcic & Wingerath (1985). The emphasis in the present research is on the effect of time dependence on the loss of stability of unsteady circular Couette flow rather than on the details of any eventual equilibrium structure. Experiments by Cole (1976) demonstrate that the *onset of instability* in the steady case is independent of apparatus length for large-enough aspect ratio. A basic assumption in the present experiments is that this same behaviour holds for a time-dependent basic state.

The experimental apparatus and procedures are discussed in §§2 and 3. The experimental results are presented in §4. These results show that, for the type of time-dependent basic states considered here, significant enhancement of stability is observed. For particular cases treated by Eagles (1977), Neitzel (1982) and Chen *et al.* (1985), this enhancement is far greater than that based on the instability criteria that they considered.

2. Experimental apparatus and instrumentation

2.1. *Taylor–Couette apparatus*

The goal of the experiments was a study of instability in unsteady circular Couette flow; the experimental condition used to determine instability is discussed below. For a fixed outer cylinder the basic state of interest is driven by a time-varying angular speed $\Omega(t)$ of the inner cylinder. This condition can be generated in an appropriately modified Taylor–Couette apparatus. The geometry of a Taylor–Couette apparatus is specified by the radii a and b ($b > a$) of the cylinders and the height h of the test section. The corresponding dimensionless parameters are the radius ratio $\eta = a/b$ and the aspect ratio $\Gamma = h/(b-a)$. The present experiments were performed in an apparatus with $b-a = 1$ in., (nominal) $\eta = 0.5$ and $\Gamma = 102.5$. The apparatus has a vertical test section and was originally designed to study the stability of spiral Poiseuille flow; details of its construction may be found in Takeuchi & Jankowski (1981). While the axial-flow capability of the apparatus is not of direct interest in the present research, it is useful in helping to control the temperature variation over the test section and was also used to eliminate vestiges of vortex structures from run to run.

Since the apparatus allows an axial flow through the annulus, the boundary conditions imposed at the ends of the test section are not the fixed-end conditions often employed in studies of the stability of circular Couette flow (e.g. Burkhalter & Koschmieder 1973). At the bottom of the test section there is a step change to an annulus with $\eta < 0.5$. This condition suppresses vortex propagation because the flow in an annulus with $\eta < 0.5$ is *more* stable than the flow in one with $\eta = 0.5$. The idea of stabilization by an axial variation in η has been recently used by Cannell, Dominguez-Lerma & Ahlers (1983). At the top of the test section, the annulus changes to a *fixed* shaped inlet annulus. The experiments were performed with fluid in this section so that the upper boundary of the test section was a liquid–liquid interface. The corresponding boundary condition is thus similar to the commonly used (e.g. Cole 1976) free-surface boundary condition. Flow visualization showed no evidence of vortex propagation from the ends of the apparatus, rather the onset of instability is the result of spontaneous growth of random disturbances. This conclusion is supported by the *quantitative* results to be presented in §4. Thus, the conditions in the present experiments seem to mirror closely the assumptions of classical hydrodynamic-stability theory.

A variable-speed d.c. motor is used to drive the inner cylinder through a timing belt. In steady operation, speed regulation is held to $\pm 0.5\%$ by a feedback control system. For the unsteady experiments the desired form for Ω was achieved by using a calibrated open-loop control system, which provided the necessary time-dependent signal to the motor. The entire drive process was controlled by a DEC MINC 23 minicomputer. Unsteady operation increased the maximum error between the desired Ω and the actual Ω to approximately 0.7%. Calibrations were performed regularly to ensure this accuracy.

A silicone oil (Dow Corning 200 fluid) with a nominal ν of 10 centistokes was the working fluid. The actual value of ν depends on the fluid temperature; this dependence was measured to an accuracy of 0.1% over the range of operating temperatures. Fluid temperatures were measured at the top and bottom of the test section using ASTM thermometers with a smallest scale division of 0.05 F°. The average of these temperatures, which differed by at most 0.1 F°, was used to calculate ν. The estimated uncertainty in ν is 0.2%. The maintenance of thermal equilibrium between the apparatus and the working fluid was assisted by the circulation of temperature-controlled fluid through the test section between experimental runs. The circulated fluid was kept at the appropriate temperature by a refrigerated circulating bath which was connected to the system sump. The physical size of the apparatus makes an external temperature-controlled bath impractical.

Initially, for flow visualization, the fluid was seeded with aluminium flakes with a maximum dimension of 40 μm. The quantitative measurements, made using a laser-Doppler velocimeter (LDV), were performed after the fluid had been filtered to remove particles larger than 4 μm. The concentration of the particles was approximately 0.005% by volume. The measured value of ν was unaffected by the presence of flakes.

2.2. *Condition for instability*

The *theoretical* basic state of interest is a pure-swirl flow. In the experiments this condition is very nearly satisfied, evidence for this being provided by measurements of the steady azimuthal component of velocity that are in good agreement with the exact solution for circular Couette flow. Thus, since the expected instability is in the form of a Taylor vortex, the presence of a non-azimuthal velocity component serves as an indicator of onset. Of the two possibilities the axial-velocity component is simpler to measure. In theory, then, the experimental condition for instability defines the onset time for instability t_0 as the smallest time for which $w(t) \neq 0$, where $w(t)$ is the measured value of the axial-velocity component at a given location in the annulus. Practically, a somewhat different condition is required because of the 'noise' in the measurement of $w(t)$; the condition actually used is of the form

$$|\langle w(t_0)\rangle| = M, \tag{2.1}$$

where $\langle w(t)\rangle$ is a suitable time-average of $w(t)$ and M is related to the sensitivity of the measuring system. For convenience, time is scaled by the diffusion time $(b-a)^2/\nu$ (nominally 64.5 s) and w is scaled by $\Omega_c(b-a)$, where Ω_c is the critical angular speed for *steady* circular Couette flow according to linear-stability theory. Specifically, for $\eta = 0.5$, Ω_c is given by (DiPrima & Swinney 1981)

$$\frac{\Omega_c(b-a)^2}{\nu} = 68.18. \tag{2.2}$$

The measurement of $w(t)$ was accomplished using a TSI single-component LDV; Doppler signals obtained in an off-axis, back-scatter mode were analysed using a 2 ns counter processor (TSI system 1980A). Since the measurement of near-zero velocities was necessary, a Bragg cell was included in the LDV system. The decision to operate in an off-axis mode was made to minimize the influence of 'velocity-gradient broadening' (Durst, Melling & Whitelaw 1981) on the measured values. The measuring volume, which has a nominal length of 2 mm, was effectively reduced in size by using a 0.2 mm mask to restrict the region of the measuring volume from which scattered light is collected. The linear dimension in the radial direction of the *effective* measuring volume is therefore of the order of 1 % of the gap. The entire LDV system was mounted on a three-dimensional traversing mechanism so that changes in the position of the measuring volume were easily made. The uncertainty in the axial position was due solely to the inaccuracy of the traversing mechanism; the difference between the indexes of refraction of the silicone oil and the glass outer cylinder added to the uncertainty in the radial position. These uncertainties were, at most, respectively 1 % and 3 % of the gap. Data from the counter were collected on a digital oscilloscope (Nicolet series 2090, with 12-bit resolution), allowing real-time monitoring of the results. Data were then transferred through an IEEE-488 bus to the computer for processing.

3. Experiments

3.1. *Procedure*

With experimental conditions as constant as possible a series of preliminary flow-visualization experiments demonstrated that both the time of onset of instability and the location at which disturbances were first noted showed a significant *random* variation from run to run. Thus it seemed necessary to perform an ensemble of experiments for each case of interest.

An important element in the experiments is the establishment of the basic state. Each experiment was begun from a state of inner-cylinder angular speed corresponding to

$$\Omega(0) = K_I\,\Omega_c, \tag{3.1}$$

where K_I is a prescribed constant ($0 < K_I < 1$). This state was ensured by requiring the inner cylinder to rotate at $\Omega(0)$ for a sufficient period of time. The starting parameter K_I is one of the parameters varied during the experimental programme.

After the initial basic state is established the computer begins to increase the inner-cylinder angular speed according to the prescribed $\Omega(t)$; this continues until a pre-determined final speed is reached and held. Simultaneously, LDV measurements of $w(t)$ are recorded on the digital oscilloscope, leaving the computer free to perform its speed-control duties. The experiment continues until a total of 1024 data points are recorded; in all cases, this is sufficient to include all events of interest. After the completion of a run the data are transferred from the oscilloscope to the computer and stored on a floppy disc for subsequent processing. Fluid is then circulated through the test section for a prescribed time before another run is initiated.

The procedure just described does, in fact, provide a (discrete) time history for $w(t)$. A suitable data-processing scheme can thus determine t_0 according to (2.1). This was originally done for several variations of the system parameters. However, during the course of a particular run, the laser was inadvertently left off until after the start of the inner-cylinder acceleration. The result of this run was a significantly longer onset time than previous runs of the same type. Several test runs consistently revealed this same effect. It was also noted that the instability was initiated near the location where the laser beams penetrated the flow field. Based on these observations it was obvious that the presence of the laser beams was affecting the stability of the flow. To account for this 'laser effect' a modification of the original experimental procedure was necessary to allow the determination of a measurement-independent onset time.

Since the LDV is normally considered to be an non-intrusive measuring device, the existence of the 'laser effect' was initially surprising. However, a reasonable explanation for it can be based on the special nature of the flow situation. The basic state of (nearly) pure swirl means that the same fluid is continually swept around the annulus through the stationary laser beams and, as a result, absorbs energy during the time prior to the onset of instability. This is in contrast to most flow situations in which the fluid passes through the LDV measuring volume only once. Because of the large Prandtl number of silicone oil (approximately 115) little of the absorbed energy is diffused to the surrounding fluid. The resulting heating decreases the fluid density locally and gives rise to natural convection (Boyd & Vest 1975, 1981). It is reasonable to postulate that the net result of this local convection is a disturbance 'source', in the form of a time-dependent axial motion, which triggers instability earlier than the disturbances provided by the apparatus.

A crude calculation shows that the *maximum* temperature rise in a 'ring' of fluid continually passing through the LDV measurement volume, a spheroid with a major axis of approximately 2 mm and 1 mm minor axis, is approximately 2 F°/min. This result is based on the assumptions that the fluid absorbs all of the rated laser-power output of 15 mW and diffuses no energy to the surrounding fluid. In order to determine a more reasonable estimate it was necessary to consider the amount of absorption by the fluid. The absorption is characterized by an absorption coefficient α which is defined by

$$I = I_0 \mathrm{e}^{-\alpha x}, \tag{3.2}$$

where I_0 is the original intensity of the laser beam and I is the intensity after traversing a distance x through the fluid. Since the value of α for silicone oil was not available, a simple experiment was performed to determine an approximate value. The experiment measured, using a photometric detector, the intensities of the laser light passing through an empty Pyrex beaker and the same beaker filled with silicone oil. Using the measured intensities in (3.2) yielded an estimate for α of 0.026 cm^{-1}.

Because the seeded fluid was used in this experiment some scattering of the laser light occurred. Thus the experiment actually measured the extinction of the laser beam, which can be considered as the maximum possible absorption (i.e. extinction = scattering + absorption) (van de Hulst 1957). Using (3.2) a more reasonable estimate of the temperature rise in the 'ring' of fluid at the usual radial position of the LDV measurement volume is approximately 0.01 F°/min. Thus, given the duration of the experiments, this effect can introduce a significant local temperature rise and a resulting disturbance 'source'.

Based on the above explanation for the 'laser effect', it was decided to minimize the time period over which the fluid was heated by the laser; the possibility of minimizing this heating by reducing the intensity of the laser is discussed below. The major modification to the original experimental procedure was the addition of a 'laser-delay' time t_L that was the *independent* time at which the laser was activated. Of course, the goal of the new experimental procedure was to determine an onset time that was independent of t_L. At t_L the internal clock of the laboratory minicomputer provided a signal for the manual opening of the shutter on the laser. Since there is an absolute error in this process, the uncertainty in t_L is largest at small t_L (approximately 1.5 %); the uncertainty decreases to about 0.5 % at the largest t_L. With an appropriate data-processing procedure (see §3.2), it was possible to determine a sequence of t_L-dependent onset times. The determination of a *measurement-independent* onset time required an extrapolation from this sequence to the condition $t_L = t_0$, i.e. to the limiting case in which the LDV is activated at the onset time. The net result is the need for additional ensembles of experiments. The extrapolation procedure is discussed in §4.2.

While the results of §4 were obtained using the above procedure, an alternate procedure in which the laser beam is attenuated to reduce its intensity may also provide a useful way to eliminate the influence of the measuring system. In order to test this idea several experiments were performed with the laser intensity attenuated by optical, neutral-density filters. Because the original signal-processing equipment was not capable of accurately monitoring the resulting weaker signals from the LDV measurement volume, the addition of an amplifier to the LDV signal processor was necessary. The results of these preliminary experiments showed that, for a given t_L, t_0 increased as I_0 decreased, but did not exceed the measurement-independent t_0 obtained by using an unattenuated laser beam. This supports the earlier physical argument on the reason for the 'laser effect'. However, even with reduced laser intensities, a dependence on t_L remained, although it was not as large as that observed without the beam attenuation. Hence, this alternate procedure would also require an extrapolation, but to the limiting case $I_0 = 0$, to obtain a measurement-independent onset time.

3.2. *Data processing*

The difficulties in processing the data to determine t_0 are obvious from an examination of a typical data set shown in figure 1. One general feature of the data is a slight shift in $w(t)$ away from zero during the time interval prior to onset. The likely source for this condition is the fact that, in spite of efforts to equalize the temperature of the apparatus to that of its surroundings, the air temperature in the laboratory was usually slightly below (at most 0.3 F°) that of the fluid (with a 0.25 in. thick Pyrex cylinder separating the two). The result is a small radial temperature gradient with colder, and hence denser, fluid near the outer wall of the annulus. Thus a slight downward axial velocity is induced near the outer wall with a corresponding upward

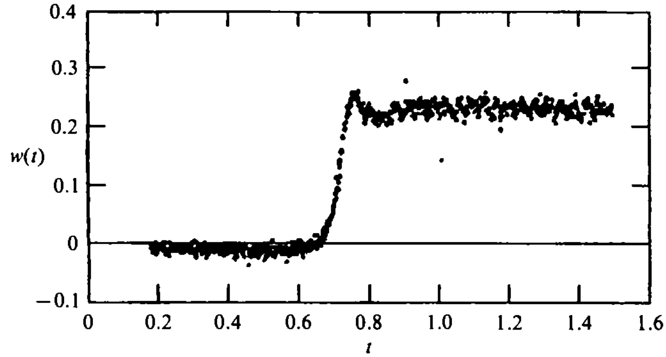

FIGURE 1. Typical raw-data set, showing a discrete time history of the axial-velocity component $w(t)$.

velocity near the inner wall. There is no theoretical evidence on which to base an estimate of the importance of this effect; however, experiments on the stability of *steady* circular Couette flow with a radial temperature gradient (Sorour & Coney 1979) support the conclusion that the small radial temperature gradients of the present experiments have a negligible effect on the *condition for the onset of instability*. However, this weak convective effect does provide a 'disturbance field' in the test section. The 'laser effect' presumably intensifies this field locally. Disturbances grow when the basic state becomes sensitive to this field (and other unknown disturbances).

The first step in the data-processing procedure is the rejection of data points which lie outside a selected band about the running average of the data and their replacement by the running average at the time in question. Typically, the use of 40 points to compute the average results in, at most, 5–10 of the 1024 data points being rejected for a given run. Since the axial-velocity component should be zero for circular Couette flow prior to onset, the data were next shifted to achieve this condition (approximately). A crude selection algorithm chooses a conservative lower bound t_0^* to t_0. The shift velocity w_s, which is the average of $w(t)$ over the time interval $t_L \leqslant t \leqslant t_0^*$, is then calculated. The magnitude of w_s is, at most, of the order of 0.01, while its sense can be upward or downward. However, when the LDV measuring volume was near the outer wall w_s was usually negative; conversely, w_s was usually positive when measurements were made near the inner wall. This evidence tends to support the earlier argument on the presence of a small radial temperature gradient. Moreover the magnitudes for w_s are similar to values determined from the exact velocity profile induced by a radial temperature gradient in a tall vertical annulus (Choi & Korpela 1980). There is no obvious dependence of w_s on t_L.

After w_s is calculated the data-reduction process shifts the data for $w(t)$ by w_s, yielding a new axial velocity

$$W(t) = w(t) - w_s; \tag{3.3}$$

the average value of $W(t)$ is zero prior to t_0^*. Finally, the determination of t_0 requires the application of (2.1) to $W(t)$ so that choices for $\langle W(t) \rangle$ and M must be made. It is clear that M must reflect the scatter of the data; the standard deviation in $W(t)$ over the interval $t_L \leqslant t \leqslant t_0^*$ is thus an obvious choice for M. Given the nature of the data for $w(t)$, a 'central running average' for $W(t)$ defined by

$$\langle W(t_i) \rangle = \sum_{i-N}^{i+N} \frac{W(t_j)}{2N+1}, \tag{3.4}$$

where i locates a discrete point on the data time axis, provides a realistic basis for the determination of t_0. The onset time t_0 is defined as the time t_i at which

$$|\langle W(t_i)\rangle| = M. \tag{3.5}$$

Typically $N = 20$ was used, but t_0 was fairly insensitive to N over the range $10 \leqslant N \leqslant 30$. The entire data-reduction process was performed by the laboratory minicomputer.

4. Discussion and results

Experiments were performed for three different inner-cylinder accelerations, which will be designated Series I, II and III. The specific formulas are:

(I) $$\frac{\Omega(t)}{\Omega_c} = K_I(1+\epsilon t), \tag{4.1a}$$

(II) $$\frac{\Omega(t)}{\Omega_c} = K_I + \epsilon t, \tag{4.1b}$$

(III) $$\frac{\Omega(t)}{\Omega_c} = K_I + (K_F - K_I)\tanh\left(\frac{\epsilon t}{(K_F - K_I)}\right), \tag{4.1c}$$

where ϵ and K_F are constants. Series I essentially employs the form used by Eagles (1977) and Neitzel (1982); note that the ramping rate is a function of K_I and hence the initial condition. For Series I and II the ramping ceases when Ω reaches a pre-determined final value $K_F\,\Omega_c$. In these cases K_F was chosen so that the final speed was greater than the instantaneous speed at which the onset of instability was noted. Either $K_F = 2.0$ or 2.5 was used depending on which gave the desired condition. Thus, the onset was unaffected by K_F although it did influence the final equilibrium amplitude. This is in contrast to the experiments of Chen *et al.* (1973) in which onset was, in most cases, observed after the final angular speed had been attained. The smooth variation of (4.1*c*) was chosen to provide results that can be compared with some current analytical work and was the form used by Chen *et al.* (1985). Theoretically, the final angular speed for Series III is $K_F\,\Omega_c$; the experiments were terminated at $0.98K_F\,\Omega_c$, where $K_F = 2.0$. For all three Series, two values of ϵ and at least five values of K_I were considered.

4.1. *Ensembles of experiments*

The independent variables in the experiments are $\Omega(t)$, the location of the LDV measuring volume and, as explained earlier, the laser-delay time t_L. With these quantities fixed, the plan was to extract t_0 from an ensemble of five experiments. It was not always possible to perform these experiments in succession since the boundary between two vortices sometimes happened to develop near the LDV measuring volume. In such cases $w(t)$ remained small for the entire experiment and the data-reduction process was unreliable; the results from these runs were discarded.

The results of a set of five acceptable runs always revealed a significant variation in onset times. Generally this variation is much larger than the variation that can be estimated from the experimental uncertainties. This behaviour was not unexpected because of the flow-visualization experiments and because of similar observations in the unsteady experiments by Kirchner & Chen (1970) and Chen *et al.* (1973). Thus an onset time cannot, in general, be identified uniquely, but can only be assumed to

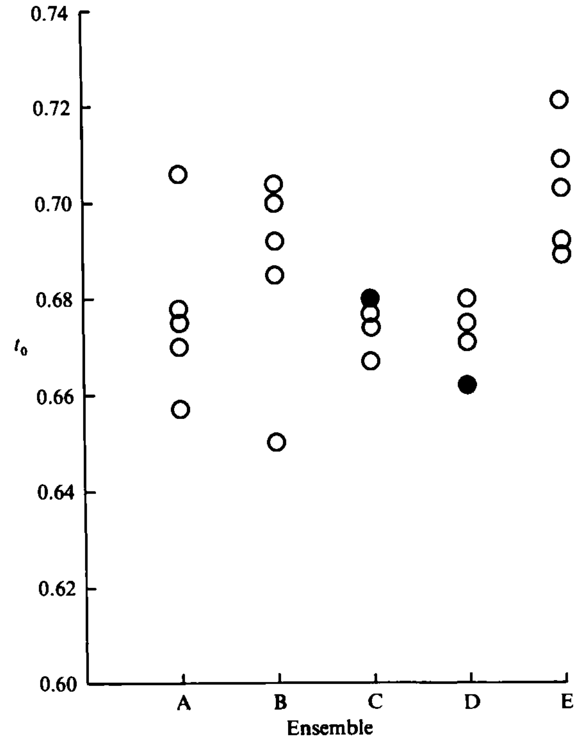

FIGURE 2. Ensembles of experiments for fixed conditions (Series II: $K_I = 0.6$, $\epsilon = 2.0$, $t_L = 0.206$). Solid symbols indicate multiple data points.

lie inside some band that is representative of the seemingly random variation in the data. The interpretation of the experiments requires some reasonable estimate of this range of onset times. Its upper bound might be associated with a linear-stability limit while the lower bound seems more likely to be related to unknown finite-amplitude effects. In order to study this effect, five ensembles of experiments were performed at different times over a one-month period. The results are shown in figure 2. They show a maximum variation of approximately 0.07 (equivalent to approximately 4.5 s), but the variation in any particular ensemble may be somewhat lower. This implies that the results from a *single* ensemble may not display a representative variation. Hence the range of onset times was chosen after examining the variations in the set of related t_L-dependent ensembles (§3.1). Typically a variation of 0.06 was reasonable.

4.2. *Influence of laser-delay time*

The physical interpretation of the 'laser effect' implies that, for a given $\Omega(t)$, the effect of t_L on t_0 should decrease as the limiting case of interest $t_L = t_0$ is approached. Clearly the onset time cannot be influenced by the LDV when onset coincides with the start of the measurement. Because of the averaging in (3.5), extrapolation must be used to study this case. The extrapolation procedure was based on results obtained at $t_L = 0$ and at three 'intermediate' values, which were chosen to divide the time interval of interest into (approximately) equal parts. Thus, for a given $\Omega(t)$, the elimination of the 'laser effect' required a minimum of 20 experimental runs. Figure 3 shows the variation of t_0 with t_L for some typical cases. As expected, the influence

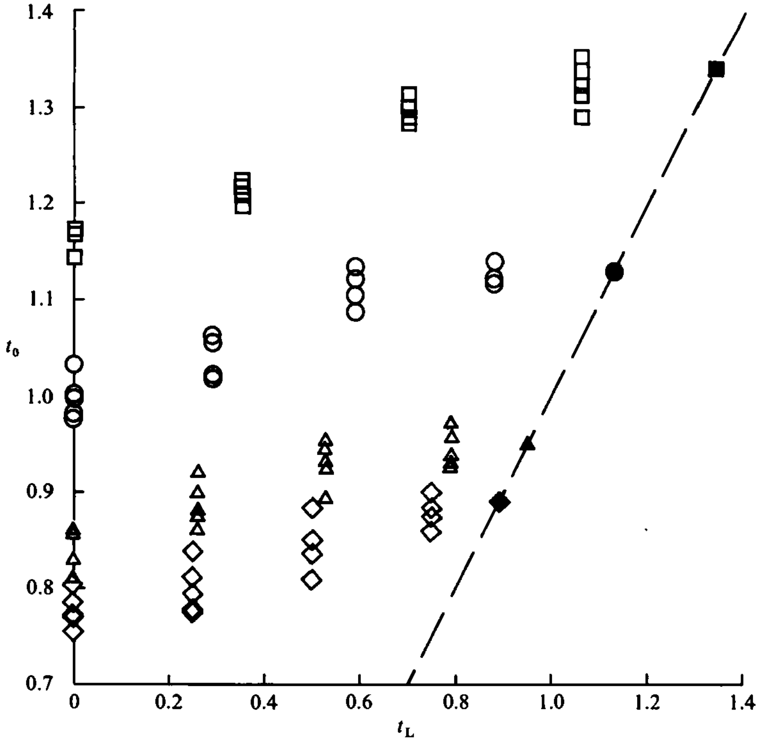

FIGURE 3. Onset times *vs.* laser-delay times for Series II ($\epsilon = 1.0$): □, $K_I = 0.4$; ○, 0.6; △, 0.8; ◇, 0.9. Solid symbols indicate average onset times at the limiting condition $t_L = t_0$.

of t_L on t_0 weakens as t_L approaches t_0. In most cases, the *overall* variation of t_0 with t_L is greater than the variation observed in an ensemble of experimental runs. For example, for $K_I = 0.4$ in figure 3, the overall variation is approximately 0.22 (which is equivalent to approximately 14 s). The influence of t_L is less for the shorter runs associated with larger K_I or larger ϵ.

Because of the variation of the data within an ensemble it did not appear reasonable to use a formal extrapolation method. Instead a smooth band was faired about the data for the four t_L-dependent ensembles and extrapolated to the condition $t_0 = t_L$. As mentioned earlier, the width of the band reflected the estimated variation in the complete data set. Examples of the average of this band at the limiting condition are shown in figure 3.

4.3. *Influence of measuring-volume position*

Several experiments were performed for four different locations, identified in table 1, of the LDV measuring volume. Some results from these experiments are shown in figure 4. They show that the differences in onset times due to changes in position are within the variation that can be expected for a single ensemble (figure 2). It can be concluded that the measured values of t_0 do not display any obvious dependence on axial or radial position. This conclusion supports the earlier observation (based on flow visualization) that the initiation of instability in the present experiments is the result of spontaneous growth of random disturbances and is not the result of vortex propagation from the ends of the apparatus. If the vortex-propagation mechanism were present, onset times would depend on the axial location of the measuring volume. The final experiments were performed at location A.

Location	Axial position (in.)	Radial position (in.)
A	28.0	1.75
B	28.0	1.25
C	46.0	1.75
D	46.0	1.25

TABLE 1. Location of LDV measuring volumes. Axial positions are measured from the bottom of test section.

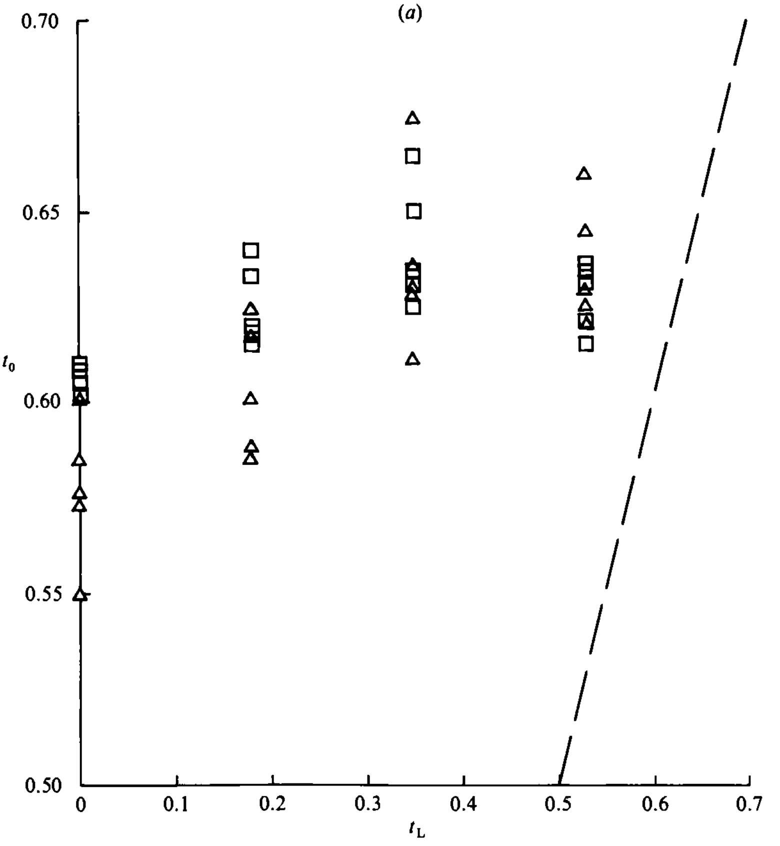

FIGURE 4(a). For caption see facing page.

4.4. *Enhancement of stability*

After the study of the individual independent variables was completed, the final sets of experiments were performed. While the direct results of these experiments are onset times, it is more informative to consider the corresponding stability enhancement $\Omega(t_0)/\Omega_c$, which is easily calculated using the average onset time in (4.1). Similarly the band about t_0, discussed in §4.1, can be used to calculate representative variations in the enhancements; these are displayed as vertical bars in the figures.

The enhancements for each Series are presented in figures 5 and 6. In each case, the flow is stabilized by the inner-cylinder acceleration, with the amount of enhancement larger for the larger accelerations. The results for Series I show a strong

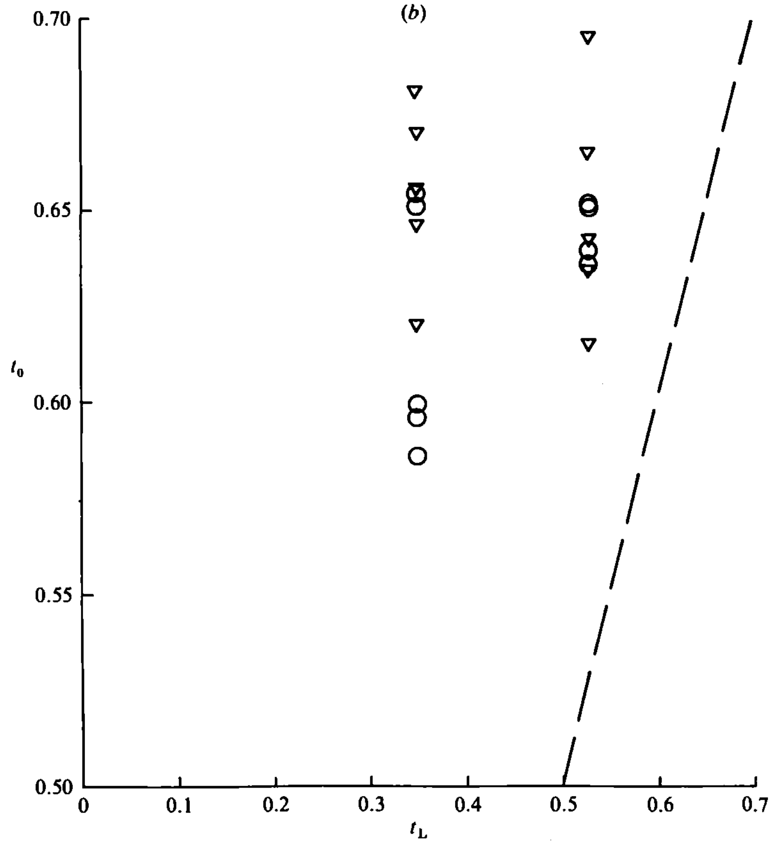

FIGURE 4. The effect of the location of the LDV measuring volume on onset times (Series II: $\epsilon = 2.0$, $K_I = 0.8$). (*a*) △, location A; □, location B. (*b*) ▽, location C; ○, location D.

dependence on K_I. This behaviour is due to the definition of the corresponding $\Omega(t)$ (4.1*a*). In contrast, K_I has only a weak influence on the enhancements for Series II and III. However, for each Series, as $K_I \rightarrow 1.0$, the enhancement should decrease since, at $K_I = 1.0$, the *initial* state is unstable and no enhancement is possible. All of the results show evidence of obeying this condition. There are no vertical bars indicating the variation in enhancement for the Series III experiments because the pertinent expression for $\Omega(t)$ (4.1*c*) is fairly insensitive to the value of t in the neighbourhood of t_0. A comparison of the results for Series II and III shows approximately the same values for the enhancements at the lower values of ϵ. The reason for this behaviour may be traced to the fact that the forms of $\Omega(t)$ for these cases are broadly similar. As predicted by Neitzel (1982) the results for Series I show the existence of an initial condition which provides a maximum enhancement.

The conclusion that stability is enhanced by inner-cylinder accelerations agrees with the available predictions. However, the experimental enhancements are much larger than the predicted values. For $\epsilon = 1.6$ in Series I, the absolute criteria of Eagles (1977) (whose definition for ϵ is a quarter the value used here) gives an enhancement of approximately 1.07; Eagles' treatment of initial conditions is discussed by Neitzel (1982). The energy-stability results of Neitzel for this case (his figure 7), which must lie below the linear-theory predictions, show a dependence on K_I similar to that shown

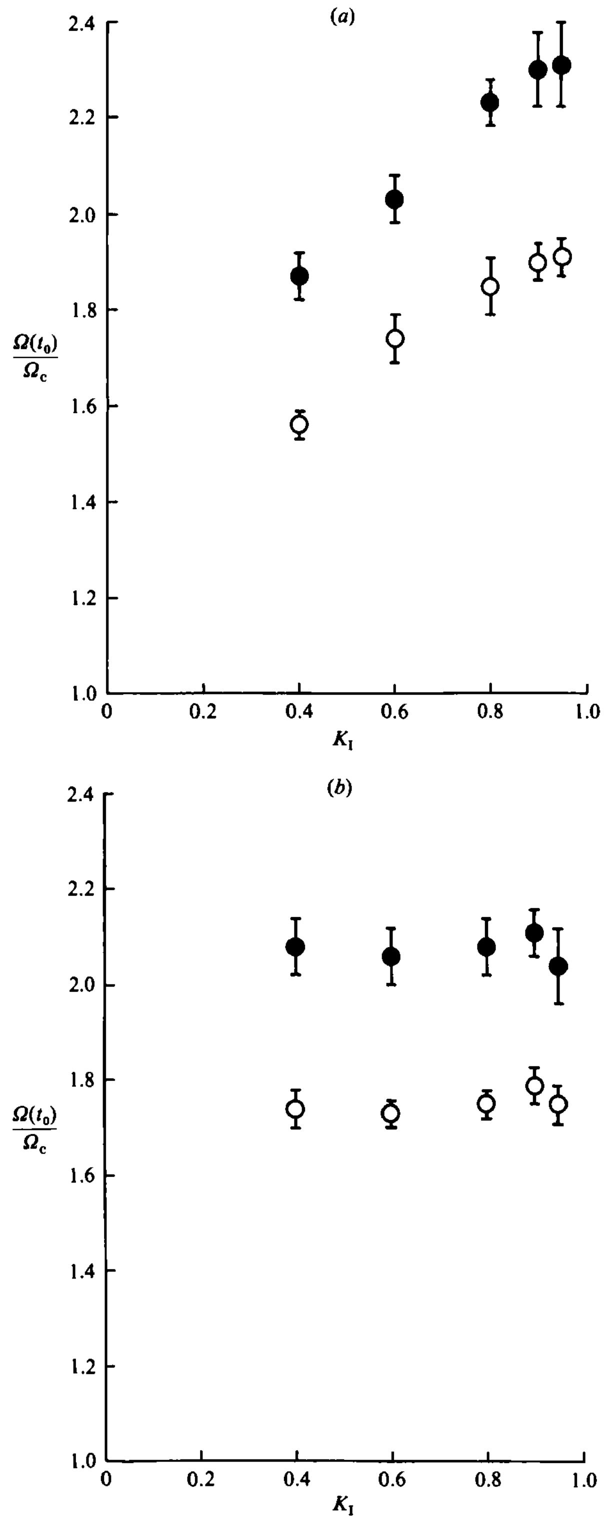

FIGURE 5. Experimental enhancements in terms of K_I. (a) Series I: ○, $\epsilon = 1.6$; ●, 3.2. (b) Series II: ○, $\epsilon = 1.0$; ●, 2.0.

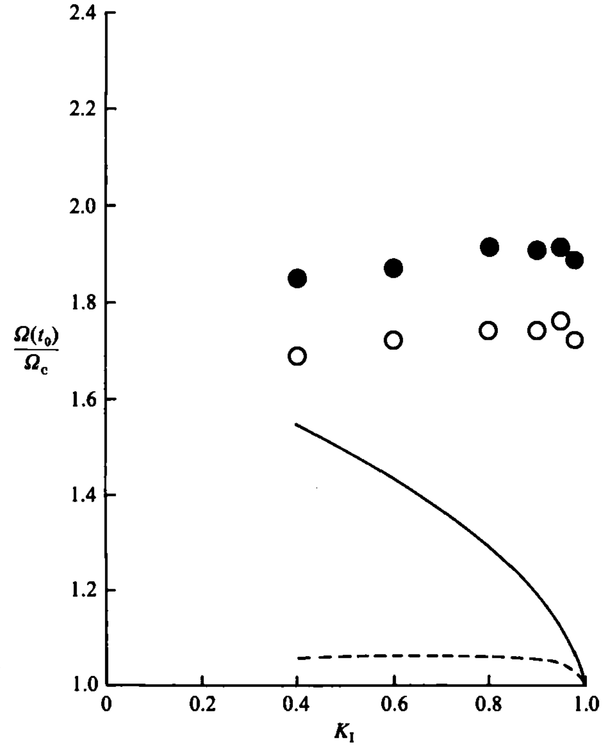

FIGURE 6. Experimental and predicted enhancements for Series III in terms of K_I: ○, $\epsilon = 1.6$; ●, 3.2. The solid line is the marginal-stability IV prediction for $\epsilon = 1.6$; the dashed line is the absolute-stability IV prediction for $\epsilon = 1.6$ (Chen *et al.* 1985).

in figure 5*a*, with a maximum enhancement of about 1.04. The recent calculations of Chen *et al.* (1985) concentrated on Series III with $\epsilon = 1.6$. Their IV results for the absolute and marginal-stability criteria are included in figure 6. The marginal criterion gives larger enhancements with values somewhat closer to the experimental values, but it also shows a definite dependence on K_I in contrast to the behaviour of the experimental results.

There are several factors which may contribute to the lack of close agreement between theory and experiment. One possible reason for the discrepancy is that the value of ϵ used in the computations is so large that the assumption of slowly varying flow is violated. The (limited) agreement between the computed results of Chen *et al.* (1985), based on the WKBJ and IV approaches, implies that this is probably not the case, however, since the IV approach does not require the flow to be slowly varying. Another obvious thought is that the growth of small disturbances occurs prior to the experimentally determined onset time, but is undetectable owing to the 'noise' in the measurements (figure 1). A more-refined measurement to test this possibility is a matter of considerable difficulty, if it is even possible.

Finally some useful ideas are suggested by Liu & Chen (1973) in a paper which considers a nonlinear IV calculation for an impulsively started inner cylinder. Instability occurs when the transfer of kinetic energy from the basic state to the disturbance is larger than the dissipation of energy; the disturbance kinetic-energy equation for unsteady circular Couette flow is provided by Neitzel (1982). In a linear theory the axial dependence of the disturbance takes a particular periodic form. Since the final equilibrium state has this same form, Liu & Chen argue that the result is an 'efficient' transfer of energy to the disturbance, and hence a reduced onset time. Near the onset conditions in the present experiments the varying axial dependence of the disturbance thus implies that an onset time longer than predicted by linear theory should be expected. In addition, the nonlinear calculations of Liu & Chen show that there are significant differences in the onset times for different initial conditions. In particular, random initial conditions gave an onset time much longer than that given by linear theory. Such conditions seem more appropriate for comparison to an experiment. A reasonable overall conclusion is that differences between the available mathematical models and the present experiments preclude, at the present time, more than the qualitative agreement discussed earlier.

It is likely that the use of an instability criterion in which the disturbance energy is allowed to grow to some arbitrarily chosen multiple of its initial value would yield better agreement with the experimental results. This conclusion is supported by the work of Chen & Kirchner (1971) and Chen *et al.* (1973). However, the arbitrariness of such a criterion makes this an undesirable theoretical approach.

The most interesting features of these experimental results are the substantial enhancements noted for every Series and the variation of the results within each experimental ensemble. A physical explanation for the enhancement and its dependence on ϵ has been given by Neitzel (1982). The existence of the observed variations implies that it may be useful to examine the behaviour of some global characteristic of the flow, such as torque, instead of the evolution of disturbances at particular locations. This has been done in experiments involving unsteady convection; references may be found in a paper by Jhaveri & Homsy (1982). Clearly, the theoretical prediction of the observed phenomena is quite difficult and is likely to require a nonlinear theory. Moreover, stability theories commonly consider the fate of a disturbance imposed at an initial time while, in unsteady experiments, unknown disturbances are imposed continuously during the entire time period of interest. (An example is the 'laser effect' of the present experiments.) There has been some recent work on unsteady thermal convection (Jhaveri & Homsy 1980, 1982) which models this behaviour. The close connection between the stability problems for thermal convection and circular Couette flow suggests that such a theoretical approach may explain some of the results observed in the present experiments.

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